Physics Factsheet

September 2000 **Number 05**

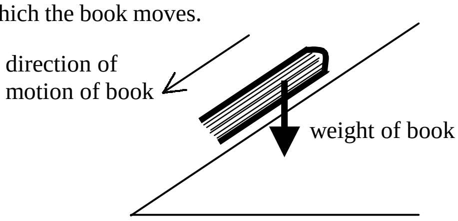
Work Energy & Power

1. Work

If a force acts on a body and causes it to move, then the force is doing **work**.

> ***W = Fs***
> *W = work done (J)*
> *F = force applied (N)*
> *s = distance moved* ***in the direction of the force*** *(m)*
>
> *This gives rise to the definition of the* ***joule****:*
> ***one joule of work is done when a force of 1 newton moves a body***

The above equation only works for a **constant** force; see the box for what happens with non-constant forces. Sometimes, the force causes something to move in a direction that's not the same as the direction of the force. The diagram below shows a book lying on a tilted smooth surface. If the surface is smooth enough, the book will slide down. The force causing the book to do this is the book's **weight**. The weight of the book does not act in the direction in which the book moves.



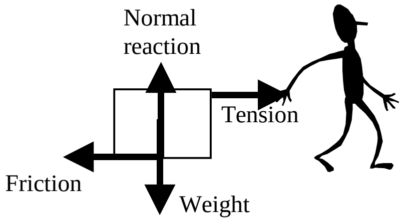
In this case, the equation is:

> *Work done by a force = distance moved × component of force in that direction*
>
> *Or, to save time resolving:* ***Work done = Fs cosθ****,*
> *where θ is the angle between the direction of the force and the direction of motion.*

Work is a **scalar** quantity, since it does not depend on the actual directions of the force and the distance moved, but on their magnitudes and the angle between them.

What happens if the object moves at right angles to the force, or in the opposite direction?

If you are dragging a box along the floor using a rope, then there are other forces on the box besides you pulling it:



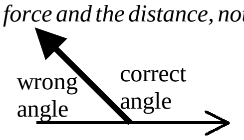
No work is done by the weight and normal reaction forces, because they are **perpendicular** to the direction of motion. This ties in with the equation W = Fs cosθ, since in this case θ is 90°, and cos90° = 0.

The friction is "trying" to stop the motion of the box. This means it does a **negative amount of work** on the box. (it can also be said that **the box does work <u>against</u> friction**). How does this tie in with the equation W = Fscosθ ?

Because the Friction force and the direction of motion are pointing in opposite directions, the angle between them is 180°.

Since cos180° = -1, this gives a negative value for work done.

> ***Tip:*** *When you are using the equation W = Fs cosθ, make sure θ is the angle between the directions of the force and the distance, not just between the lines representing them.*



What happens if the force is not constant?

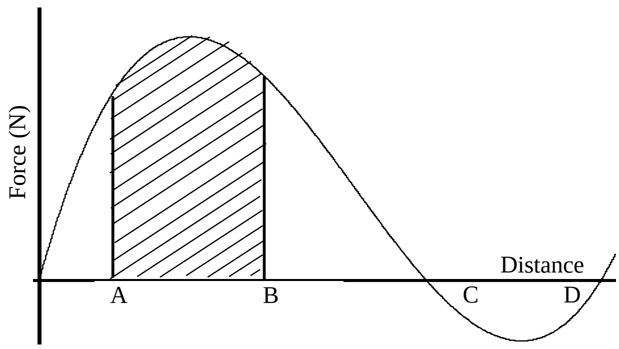
To find the work done by a non-constant force, there are two options:

- use the average force in the normal equation
- obtain the work done from a graph

To find out the work done from a graph, you draw a graph of force (on the y-axis) against distance (on the x-axis). The area **under** the graph is the work done.

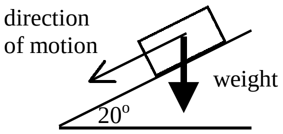
In the graph shown right, the shaded area gives the work done by the force in moving from point A to point B.

Note that between points C and D, the force becomes negative – meaning it is in the opposite direction – so the work done by the force will be negative (or equivalently, work will be done against the force).



Example 1. A smooth plank of wood has one end fixed to the floor and the other resting on a table, so that the wood makes an angle of 20^o with the horizontal floor.

A book of mass 0.9kg is held so that it rests on the plank. It slides down 40cm before coming to rest at the bottom of the plank. Find the work done by the book's weight. Take g = $10ms^{-2}$



To apply the formula W = Fs cosθ, we need to know the angle between the direction of motion and the force. By using angles in a triangle sum to 180^o, we find it is 70^o.

So $W = 0.9 \times 10 \times 0.4 \times \cos 70^o = 1.23J$

(Alternatively, we could have found the component of the weight in the direction of the motion, which would be $0.9 \times 10 \times \sin 20^o$, then multiplied it by the distance. This gives the same result.)

2. Energy

Energy is the ability to do work

There are many different forms of energy – chemical, electrical, heat, light, sound, nuclear, kinetic and potential. In fact, almost all of these are types of **kinetic energy** or **potential energy**.

Kinetic energy
- *is the energy a body possesses because it is moving.*
- *It can be defined as the work required to increase its velocity from zero to its current value.*
- *k.e. = ½ mv^2, where m = mass (kg) and v = speed (ms^{-1})*

Potential energy
- *is the energy a body posseses due to its position (or the arrangement of its parts).*

Gravitational potential energy
- *can be defined as the work required to raise the body to the height it is at.*
- *g.p.e. = mgh, where,*
  m = mass (kg)
  g = acceleration due to gravity (ms^{-2}),
  h = height of centre of mass above some set reference level.
- *Note*
  - *it is arbitrary where we measure the height from – we can choose whatever level is convenient*
  - *if a body is **below** this level, then h, and hence its gravitational potential energy, are negative.*

***Tip**. You may be asked to describe the energy changes involved in a process. Remember:*
- *if there is any **friction** involved, some **heat energy** will be produced*
- *if the process makes a noise, **sound energy** will be produced*
- ***chemical energy** includes the energy inside batteries and energy gained from food*
- *if something is falling, it will be losing **potential energy***
- *if something is slowing down or speeding up, it will be losing orgaining **kinetic energy***

Typical Exam Question

(a) Is work a vector or a scalar quantity? Explain [2]

(b) A child of mass 26 kg starting from rest, slides down a playground slide. What energy changes occur during the descent? [3]

(a) Work is a scalar ✓ It is independent of the directions of the force or distance moved, and just depends on their magnitude.✓

(b) Gravitational potential energy ✓ changes to kinetic energy,✓ heat ✓

Deriving the formulae for kinetic and potential energy

Kinetic energy

Let us consider a constant force F acting on a body of mass m, so that it moves a distance s and increases its speed from 0 to v.

Then we know that the work done by the force is given by:

W = Fs

We also know, from Newton's 2nd Law, that

F = ma

where a is the acceleration of the body.

Putting these two equations together, we get

W = ma × s (1)

Now, from equations of motion, we know

$2as = v^2 - 0^2$ (since u = 0)

Rearranging this equation, we get

$as = ½ v^2$ **(2)**

Putting equation (2) into equation (1), we get

$W = mas = m \times ½ v^2 = ½ mv^2$

Gravitational potential energy

Let us consider raising a body of mass m through a vertical height h.

The weight of the body is mg, acting downwards. To overcome this, a force of equal magnitude but opposite direction has to be exerted – in other words, a force of mg upwards.

The work done by this force = force × distance moved = mgh.

3. Conservation of energy

*The **principal of conservation of energy** states that:*
Energy may be transformed from one form into another, but it cannot be created or destroyed.

Here, we shall be considering a special case of this, which concerns **mechanical energy** – which is gravitational potential energy and kinetic energy.

*The **principal of conservation of mechanical energy** states:*
*The total amount of mechanical energy (k.e. + g.p.e.) which the bodies in a system possess is constant, **provided no external forces act**.*

The "no external forces acting" means that:
- there is no friction
- no other "extra" force – such as a car engine or someone pushing – is involved

Weight does NOT count as an external force.

The principal of conservation of mechanical energy can be used to work out unknown velocities or heights, as shown in the examples below.

There are a number of slightly different approaches to doing conservation of energy calculations; here we will always use:
Total mech. energy at the beginning = Total mech. energy at the end.

Example 2. A ball of mass 50g is thrown vertically upwards from ground level with speed 30ms^{-1}.
a) Find the maximum height to which it rises, taking g = 9.81ms^{-2}
b) Find its speed when it is 10m above ground level
b)Give one assumption that you have used in your calculation.

a)We will take potential energy as 0 at ground level

> ***Tip**. It is helpful to make a definite decision where to take potential energy as zero, since it is then easier to make sure you always measure from the same place.*

Total mechanical energy at start $= g.p.e. + k.e.$
$= mgh + ½ mv^2$
$= 0 + ½ \times 0.05 \times 30^2$

> ***Tip**. To avoid rounding or copying errors, do not work out the actual figures until the end*

Total mechanical energy at end $= 0.05 \times g \times h + 0$

> ***Tip**: Remember the ball will rise until its velocity is zero. You often need to use this idea in this sort of problem.*

Conservation of mechanical energy gives:

$$0 + ½ \times 0.05 \times 30^2 = 0.05 \times g \times h + 0$$

$$\text{So } h = \frac{\frac{1}{2} \times 0.05 \times 30^2}{0.05 \times 9.81} = 45.87....m = 46m \text{ (2 SF)}$$

b) Total energy at start = $½ \times 0.05 \times 30^2$*, as before*
Total energy when 10m above ground = $0.05 \times g \times 10 + ½ \times 0.05 \times v^2$
So $½ \times 0.05 \times 30^2 = 0.05 \times g \times 10 + ½ \times 0.05 \times v^2$

$$\text{So } v^2 = \frac{\frac{1}{2} \times 0.05 \times 30^2 - 0.05 \times 9.81 \times 10}{\frac{1}{2} \times 0.05} = 703.8$$

$$\Rightarrow v = \sqrt{703.8} = 27 \text{ ms}^{-1} \text{ (2SF)}$$

c) Since we have used conservation of mechanical energy, we are assuming that no external forces act. So in this case, we are assuming there is no air resistance.

Example 3. A book is dropped from a height of 4 metres.
Taking g = 10ms^{-2}, find its speed when it hits the floor

a) We will take potential energy as 0 at floor level.

Total mechanical energy at start $= mg \times 4 = 4mg$
Total mechanical energy at end $= ½ mv^2$

You might be tempted to panic because we don't have a value for m. But it will cancel out, since it occurs on both sides of the equation:

Using conservation of mechanical energy:

$4mg = ½ mv^2$
So $8g = v^2$
$v = \sqrt{80} = 8.9 \text{ ms}^{-1}$

4. The Work-Energy Principle

When external forces are involved, conservation of mechanical energy cannot be used. Instead we use the **work-energy principle**.

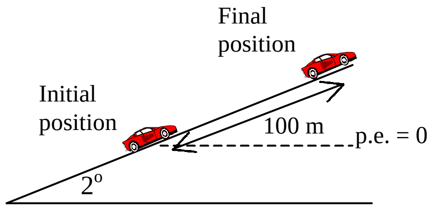
> *Work done by external force = increase in total mechanical energy + work done against friction*

The following examples show how to use this:

Example 4.
A car of mass 800kg is at rest on a road inclined at 2^o to the horizontal. Later, it is 100m further up the road, travelling at 10 ms^{-1}.
Assuming there are no frictional resistances to motion, find the average driving force exerted by the car's engine. Take g = 9.8ms^{-2}

Note: we know we cannot use conservation of mechanical energy because the car has gained both kinetic and gravitational potential energy.

Step 1. Draw a diagram, & decide where to take potential energy as zero



Step 2. Work out initial & final total energy

Total initial energy = 0

Final k.e. $= ½ \times 800 \times 10^2$

To find the final g.p.e., we need to work out the vertical height of the car above its initial position

Using trigonometry, we find $h = 100\sin 2^o$
So final g.p.e = $800 \times 9.8 \times 100\sin 2^o$

So total final energy = 67361J

Step 3. Use the work energy principle, putting in everything we know

WD by external force = inc in mechanical energy + WD against friction

There is no work done against friction, since we are told there is no frictional force.

The only external force involved is the driving force, D, which acts directly up the hill, in the same direction as the distance moved.

So we have: $D \times 100 = 67361 - 0$
So D = 673.61N = ***670N (2SF)***

> ***Exam Hints:***
> *1 Ensure you write down the equation you are using - this will probably gain you a "method" mark, even if you make a careless mistake.*
>
> *2 Do not lose out by using too many or too few significant figures - you should never use more SF than the question does*

Typical Exam Question

(a) (i) Write down an equation for W, the work done when force F moves an object a distance s, in a direction that makes an angle θ° to the line of action of the force. [1]

(ii) Use this equation to derive an expression for the gain in gravitational potential energy when a mass m is raised through a height h. [2]

(b) (i) Describe the energy changes taking place when a person jumps upwards from standing. [2]

(ii) If the person has achieved a speed of $6.1ms^{-1}$, calculate the maximum height that they can jump. [2]

(iii) What assumption have you made in this calculation? [1]

(iv) If the take off speed could be increased by 10%, would there be a 10% increase in the height jumped? Explain. [2]

(a)(i) $W = Fs\cos\theta$ ✓

(ii) Gain in GPE = work done in raising mass through height h ✓
$F = mg \quad s = h$ *and* $\theta = 0$ *as force and distance are vertical*
$W = mgh$ ✓

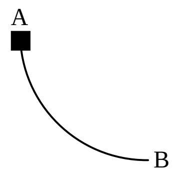
(b)(i) Chemical energy from food is changed into kinetic energy and heat in her muscles ✓
Her kinetic energy is then transformed into gravitational potential energy. ✓

(ii) Kinetic energy is transformed into gravitational potential energy:
$\frac{1}{2}mv^2 = mgh$ ✓
$h = v^2 / 2g = 6.1^2 / 2 \times 9.8 = 1.9m$ ✓

(iii) The calculation assumes that all of the kinetic energy at take off is transformed into gravitational potential energy and that there are no other energy transformations. ✓

(iv) No, since $KE = \frac{1}{2}mv^2$ *, a 10% increase in speed would produce an energy increase of more than 10%.* ✓
$mgh = \frac{1}{2}mv^2$ *so the height is proportional to the KE*
The increase in height will also be more than 10% ✓

Example 5. The diagram below shows a curved track in the form of a quarter of a circle of radius 10cm.
A small block of mass 30 grammes is held at point A, then released. Given that the average frictional force between the block and the track is 0.1N, find the speed of the block when it reaches B. Take $g = 10ms^{-2}$



Taking potential energy as 0 at B:

Initial g.p.e. $= 0.03 \times 10 \times 0.1 = 0.03J$ *Initial k.e. = 0.*
So initial mechanical energy = 0.03J
Final g.p.e. = 0 *Final k.e.* $= \frac{1}{2} \times 0.03 \times v^2$
So final mechanical energy $= 0.015v^2$

Work done against friction = frictional force × distance moved
Distance moved = quarter of circle $= \pi(0.1)^2 \div 4 = 0.0025\pi$.
So work done against friction $= 0.1 \times 0.0025\pi = 0.00025\pi$.

Now use the equation. There are no external forces acting (except friction)

$0 = 0.015v^2 - 0.03 + 0.00025\pi$

So $v^2 = \dfrac{0.03 - 0.00025\pi}{0.015} = 1.9476....$ So $v = 1.40ms^{-1}$ (3SF)

Tip *Increase in mechanical energy is* ***always***
final energy – initial energy.

Typical Exam Question

A toy car of mass 0.3kg is moving along a horizontal surface. It is travelling at $4ms^{-1}$ when it reaches the foot of a ramp inclined at an angle of 20° to the horizontal.

(a) (i) Calculate the kinetic energy of the car at the instant it reaches the foot of the ramp [1]

(ii) The car rolls up the ramp until it stops. Ignoring resistive forces, calculate the vertical height through which the car will have risen when it stops. Take $g = 9.8ms^{-2}$ [2]

(b) In practice the car only rises by 75% of this theoretical value. Calculate

(i) the energy lost in overcoming resistive forces. [1]

(ii) the average resistive force acting on the car, parallel to the slope of the ramp. [4]

(a) (i) $KE = \frac{1}{2}mv^2 = \frac{1}{2} \times 0.3 \times 4^2 = 2.4J$ ✓

(ii) $mgh = 2.4J$ ✓
$h = 2.4 / (0.3 \times 9.8) = 0.82m$ ✓

(b) (i) Energy lost in overcoming friction = 25% of 2.4J = 0.6J ✓

(ii) Frictional force × distance moved = WD against friction = 0.6J ✓
Distance moved up slope = s
Where, $s\sin 20° = (75 / 100) \times 0.82$ ✓ $s = 1.8m$ ✓
Frictional force $\times 1.8 = 0.6$ *so friction = 0.6 / 1.8 = 0.33N* ✓

5. Power

Power is the rate of doing work.
If work is being done at a steady rate, then:

$$\text{power} = \frac{\text{work done}}{\text{time taken}}$$

The unit of power is the ***watt****; 1 joule of work being done per second gives a power of 1 watt.*

Example 6. A pump raises water through a height of 4.0m and delivers it with a speed of $6.0ms^{-1}$***. The water is initially at rest. The pump moves 600kg of water every minute. Taking*** $g = 9.8ms^{-2}$***, calculate the power output of the pump. (You may assume that the pump works at a steady rate)***

First, we need to find the work done.

Every minute, the pump raises 600kg of water through 4.0m, and gives it a speed of $6.0\ ms^{-1}$
So every minute:
increase in kinetic energy $= \frac{1}{2} \times 600 \times 6^2 = 10800J$
increase in gravitational potential energy $= 600 \times 9.8 \times 4 = 23520J$
So increase in mechanical energy per minute = 10800 + 23520 = 34320J

So the pump must do 34320J of work on the water every minute.
So power of pump = 34320/60 = 572W = 570W (2SF)

Tip*. Be careful to always use SI units – in the above example, it might have been tempting to work in minutes, but seconds are what is required by the definition of the watt.*

There is also another form of the power formula which is particularly useful when working out the power of a machine - like a car or a train – exerting a **constant driving force**.

We know: work done = Force × distance moved in the direction of force.
So, if the driving force is D, we have work done = Ds
If work is done at a constant rate, and the velocity is constant we have:

$$\text{Power} = \frac{Ds}{t} = D \times \frac{s}{t} = Dv$$

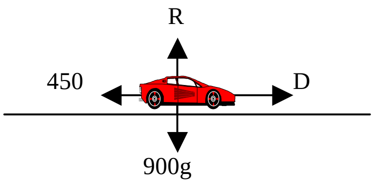
Power = Driving Force × velocity

When using the equation P = Dv

When using the equation P = Dv, you often also need to use
- resolving, to find the driving force
- the idea that if something is moving with constant velocity, the resultant force on it is zero.

Example 7. A car of mass 900kg can travel at a maximum speed of 20 ms^{-1} on a level road. The frictional resistance to motion of the car is 450N.
a) Find the power of the car's engine.

The car is travelling up a road inclined at sin^{-1}(0.02) to the horizontal with its engine working at the same rate. Find its maximum speed. Take g = 9.8ms^{-2}



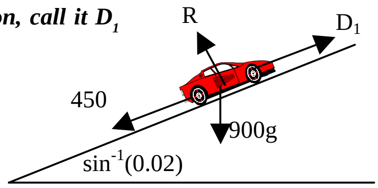
a) Step 1. Draw a diagram, showing all forces.
Step 2. Resolve in the direction of motion.
Note: if the car is at maximum speed, there is no resultant force

$D - 450 = 0 \Rightarrow D = 450$

Step 3. Use the power equation
$P = Dv = 450 \times 20 = 9000W$

b) Note that "engine working at the same rate" means "exerting the same power" ***It does NOT mean that D is the same. To avoid confusion, call it D_1***

1.



2. ***We must resolve up the slope, since that's the direction the car is moving in.***
$D_1 - 450 - 900g\,(0.02) = 0$
$D_1 = 626.4N$

3. $P = Dv \Rightarrow 9000 = 626.4v \Rightarrow v = 14$ ms^{-1} *(2 SF)*

Typical Exam Question

(a) Show that the unit of power is equivalent to that of force × velocity. [2]

(b) On a straight, level road a cyclist with a power ortput of 95 W can cycle at a maximum steady speed of 5ms^{-1}. The combined mass of the cyclist and cycle is 90kg. Calculate:
(i) The total resistive force exerted on the cyclist. [2]
(ii) Assuming that the resistive force remains the same, calculate the maximum speed the cyclist can maintain up a slope of 1 in 50. [4]

(a) Units of force × velocity = N × (m / s) ✓
Units of power = J / s = N m / s ✓
(b)(i) Force = power / speed ✓= 95 / 5 = 19N ✓
(ii) Assume V ms^{-1} is the maximum speed.
Work done in one second against resistive force = power = force × velocity = 19 × V = 19V joules ✓
Work done in 1 second raising the gpe of the cyclist = mgh = 90 × 9.8 × V / 50 = 17.6V joules ✓
Total work done per second = 19V + 17.6V = 36.6V joules
36.6V = 95 ✓ so V = 95 / 36.6 = 2.6ms^{-1} ✓

Efficiency

$$\text{efficiency} = \frac{\text{useful energy output}}{\text{total energy input}} \times 100\%$$

No real machine is 100% efficient – there is always some "lost" energy.

Of course, this "lost" energy is actually converted to another form – usually heat, generated by friction. But it is not possible to make use of it.

Questions involving efficiency may require you to:
- work out the efficiency of a machine
- work out the energy input, when you know the efficiency and the energy output
- work out the energy output, when you know the efficiency and the energy input.

Typical Exam Question

(a) When an athlete is performing press-ups, the average force in each arm is 200N. Calculate the work done by his arms during one press-up, which raises his shoulders 0.50m above the ground. [2]

(b) If the athlete can do 16 press-ups per minute, calculate the:
(i) total power output of his arms. [2]
(ii) energy input to his arms in one minute, if the overall efficiency of his arms is 20%. [2]

(a) Work = force × distance = 200 × 0.5 ✓= 100J per arm
Total work = 200N ✓
(b) (i) Power output = work done / time taken = 16×200 / 60 ✓ = 53.3W ✓
(ii)(power output / power input) × 100 = 20
pwr input = pwr output × 100 /20=53.3×100 / 20 ✓=266.5W
Energy input in one minute = 266.5 × 60 = 1.6 × 10^{4}J ✓

Questions

1. Define the *joule* and the *watt*.
2. Explain what is meant by *efficiency*.
3. Explain what is meant by the *principle of conservation of mechanical energy*, and give the circumstances in which it can be used.
4. Is work a scalar or a vector?
5. A force of magnitude 6N moves a body through a distance of 2 metres. Find the work done by the force if
   a) the distance moved is in the direction of the force
   b) the distance moved is inclined at 60° to the direction of the force.
6. A ball is thrown vertically upwards from ground level with speed 20ms^{-1}. Taking g = 10ms^{-2},
   a) Find the height to which it rises.
   b) Find its speed when it is 2m above ground level.
   In the above calculation, you have neglected a force
   c) What is the name of this force?
   d) Would you expect your answer to a) to be larger or smaller if this force were taken into account? Explain your answer.
7. A marble is dropped from a height of 80cm above the ground.
   a) Neglecting air resistance, and taking g = 10ms^{-2}, find its speed as it reaches the ground.
   The ground that the marble falls on is muddy, and the marble sinks 2cm into the mud.
   b) Given that the mass of the marble is 10g, find the average force exerted by the mud on the marble.
8. A car of mass 800kg has a maximum speed of 20ms^{-1} down a slope inclined at 2° to the horizontal. The frictional resistance to motion of the car is constant, and of magnitude 1000N. Take g = 10ms^{-2}.
   a) Find the power of the car's engine
   b) Find its maximum speed up the same hill.

Exam Workshop

This is a typical poor student's answer to an exam question. The comments explain what is wrong with the answers and how they can be improved. The examiner's answer is given below.

(a) Define:

(i) power. **[1]**

P = W/t

0/1

> Using an equation is fine, but only provided you define the symbols used in it.

(ii) the watt. **[1]**

1 watt = 1 joule per second ✓

1/1

> Although this was awarded the mark, it might have been better to say "1 joule of work is done per second" or "1 joule of energy is transferred per second"

(b) A car of mass 1200kg accelerates from rest along a straight, level road. If the car has a constant acceleration of 2.50ms^{-2}, calculate:

(i) the force causing this acceleration. **[1]**

3000N✓

1/1

> Although the candidate's answer is correct, s/he would be wise to show their working and include the formula used, to ensure full credit is gained.

(ii) the work done in moving the car the first 100m. **[1]**

300000

0/1

> Although the answer is numerically correct, the mark was not awarded because the candidate omitted the units.

(iii) the average power output of the car during the first 100m **[3]**

$s = ½ at^2$.

$100 = 1.25t^2$. $t = \sqrt{80} = 8.944271$ ✓

Power = 300000/8.944271 ✓= 33541.02 W

2/3

> Candidate was not awarded the final mark, since the number of significant figures given in the answer was far too great.

Examiner's Answers

(a) (i) Power = work done / time taken ✓

or: power is the number of joules of energy converted from one form to another in one second

(ii) Power of 1 watt is developed when 1 joule of work is done in 1 second ✓

(b) (i) $F = ma = 1200 \times 2.50 = 3000N$ ✓

(ii) Work done = force × distance = $3000 \times 100 = 3 \times 10^5 J$ ✓

(iii) Power = Work Done / time taken ✓

$s = ut + ½ at^2$ $u = 0$

So: $s = ½ at^2$

Giving time taken as: $t = \sqrt{(2s/a)}$

$= \sqrt{(2 \times 100 / 2.5)} = 8.94s$ ✓

Power $= 3 \times 10^5 / 8.94 = 3.36 \times 10^4 W$ ✓

Answers to questions

1. – 4. can be found in the text

5. a) 12J b) 6J

6. a) 20m. b) 19.0ms^{-1} c) air resistance

d) smaller, because the total mechanical energy will have decreased.

7. a) 4ms^{-1} b) 4N

8. a) 14400W b) 11.3ms^{-1}

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