



Stress and Strain in Metals

The greatest proportion of time spent studying the extension of metals (when a force is applied) is concerned with the metal in the elastic region. And we tend to specifically concentrate on the part of this region below the limit of proportionality – we look at Hooke's Law, the spring constant, and the Young modulus.

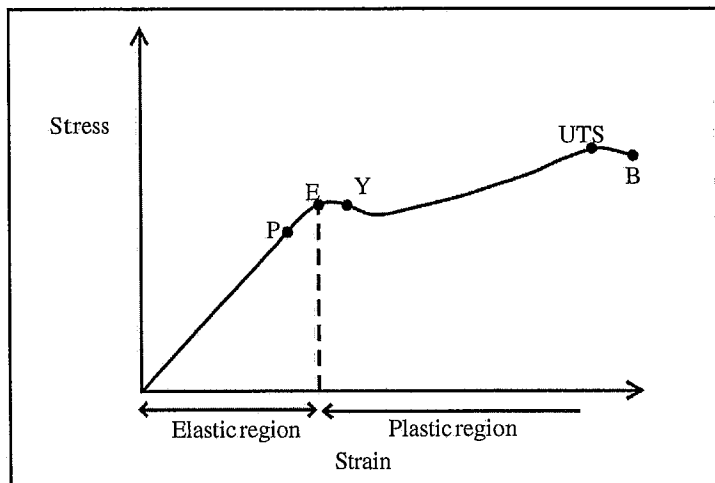
This Factsheet will look more generally at metals when they are being extended. We will look at:

- different types of metals
- atomic structure
- relevant calculations

and we will study the metals right up to their breaking point.

Stretching to destruction

As mentioned, we want to see what happens to a metal wire as it is stretched until it breaks. Here is a typical example:



The axes are "stress" and "strain".

Stress is defined as the force per unit cross-sectional area, $\sigma = F/A$ (Nm^{-2} or Pa).

Strain is defined as the extension per unit original length, $\epsilon = e/l_0$ (no units).

Key: Stress and strain are properties linked to a material, rather than to a specific sample of the material. The stress-strain graph should be the same for any thickness and length of wire made of the same material.

The graph itself is best explained by defining the labelled points:

P is the limit of proportionality. The extension up to this point is elastic, and obeys Hooke's Law. The strain is proportional to the applied stress. If the stress is relaxed, the wire returns to its starting length.

E is the elastic limit. Up to this point, the wire will return to the starting length if the stress is relaxed. However the graph is curving – Hooke's Law is no longer obeyed.

Key: Up to the elastic limit, *E*, the wire has not been permanently deformed. The implications for stored energy are discussed later in the factsheet.

Y is the yield point. The wire is now being permanently deformed, and at point Y there is a sudden increase in strain actually accompanied by a reduction in applied stress.

UTS is the ultimate tensile stress. This is not necessarily the breaking point. The wire is deforming plastically. The wire will now continue to stretch with reduced applied stress.

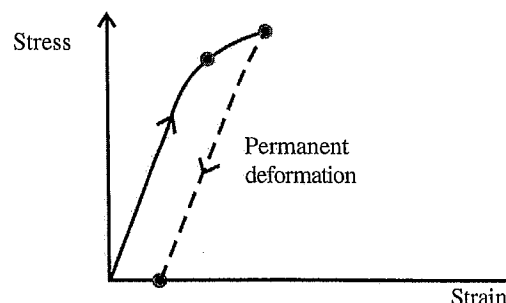
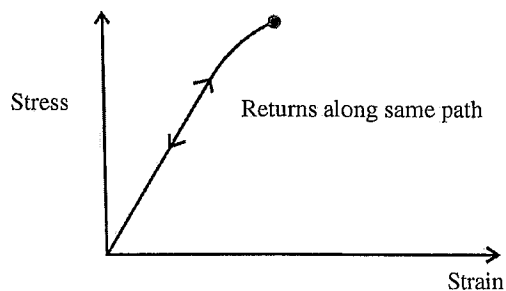
B is the breaking point. The wire breaks at a weak point. The value of the strain at this point is a property of the individual sample of wire.

Key: After the elastic limit, the extension of the wire is in the plastic region. Permanent deformation is taking place. As we will see, there are implications for energy transfer.

Example 1:

Sketch graphs showing what happens to the wire if the stress is relaxed when the wire is at its elastic limit, *E*, and when it is at its yield point, *Y*.

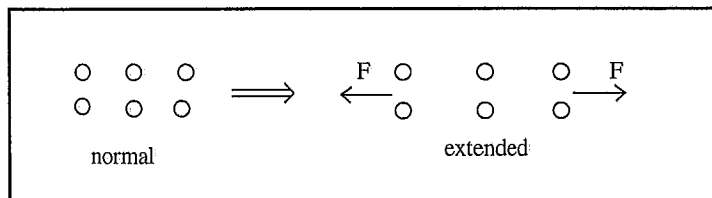
Answer:



Atomic structure explanation

Very briefly, we will look at what happens in the elastic and plastic regions.

(a) Elastic

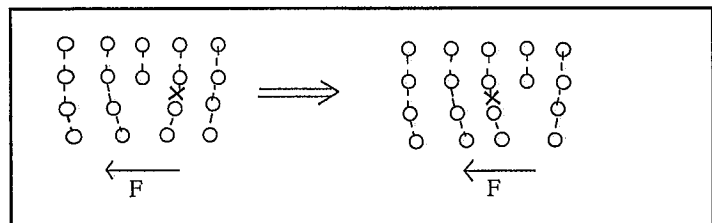


The atoms in the lattice stay in position relative to each other. However the applied force pulls them away from each other. At first the extension is proportional to the applied force. As the force is increased, the extension is no longer proportional. However the atoms still return to their original positions when the force is relaxed.

Key The energy transferred to the lattice as elastic potential energy can all be used to do work when the lattice returns to its original shape.

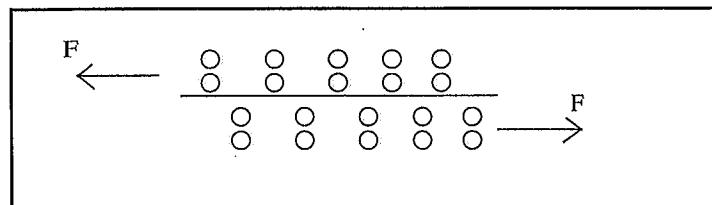
(b) Plastic

In the plastic region a number of processes occur.



At a dislocation in the crystal structure, the applied force has broken a bond causing a crystal plane to move along one position. The sample has undergone permanent deformation.

And if the applied force is increased even more, whole planes of atoms can slide past each other, greatly increasing the strain in the wire.

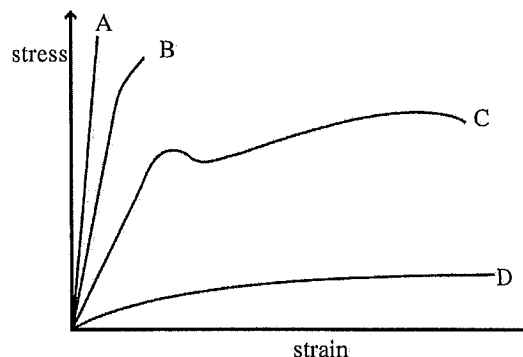


Again, the deformation is permanent.

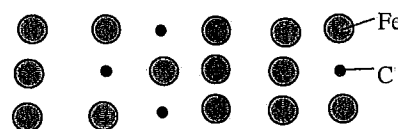
Key The energy required to cause permanent deformation cannot be regained. It will have been transformed into thermal energy.

Different types of materials

The original graph is representative of many metals. However different types of metals will give different responses when a force is applied.



A is a brittle material e.g. cast iron. It deforms elastically, but there is very little strain even for a large applied stress. Then it suddenly breaks. Cast iron has a high proportion of “impurity” atoms (carbon) introduced. The different atom sizes make plastic deformation very difficult.



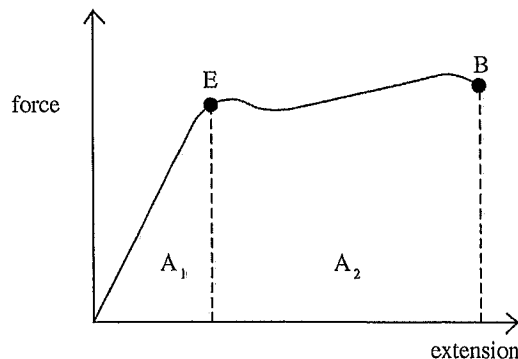
B is a strong material e.g. steel. There are fewer “impurity” atoms (carbon). More movement is possible, but it is still very difficult for plastic deformation to occur.

C is a ductile material e.g. pure copper. The lack of impurity atoms makes it much easier for atom planes to slide past each other. Most pure metals undergo plastic deformation.

D is a plastic material e.g. pure lead. It is so easy to slide the crystal planes past each other that there is very little elastic stretching.

Energy under a Force-Extension Graph

A force-extension graph is very similar in shape to a stress-strain graph. The area under the force-extension graph gives a value for the work required to stretch the wire sample.



This wire is stretched past its elastic limit, E, right through its breaking point, B.

Work done in breaking the wire, $W = A_1 + A_2$

Energy stored as elastic potential energy in the wire, $EPE = A_1$

Energy “lost” as thermal energy, $E = A_2$

Example 2:

Why is a steel wire much more dangerous when it breaks than a copper wire?

Answer:

Almost all of the work done on the steel wire has been stored as elastic potential energy (rather than transferred into thermal energy). The release of energy upon breaking can cause the steel wire to “whiplash” violently.

Example 3 :

Why is a force-extension graph slightly different in shape to a stress-strain graph?

Answer:

The strain will be proportional to the extension. However, as the wire stretches, its cross-sectional area decreases. As $\sigma = F/A$, the changing area will mean that the stress will not be exactly proportional to the applied force.

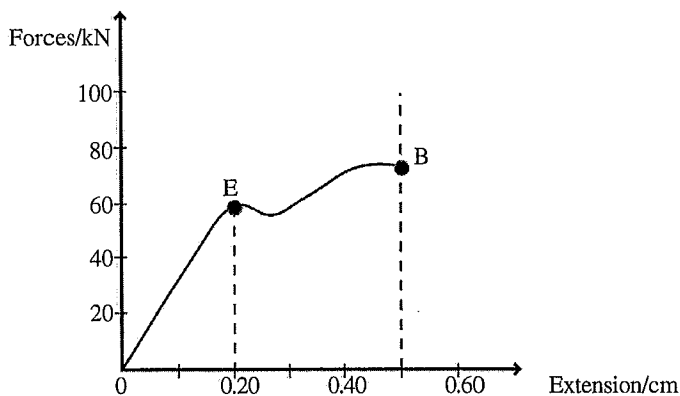
Calculations

The difficulties with calculations arise from the large values for force, combined with the small values for extension. Force may be measured in kN, and extension may be fractions of a millimetre. And the radius of the wires involved may also be less than a millimetre.

Exam Hint:- Take care with the detail of the calculations in this topic. Notice whether units are provided in N or kN (for example).

Practice Questions

1. Estimate the elastic potential energy stored in this wire before it breaks, and the energy transferred to heat in stretching it.



2. Find the maximum load that can be supported by a steel cable 1.2cm in diameter. (Ultimate Tensile Stress = 500MPa)
3. A hammer thrower swings a hammer (mass 6.8kg) in a horizontal circle of radius 1.65m. The speed of the hammer is 6.5ms⁻¹. The steel wire has a radius of 2.5mm.
- (a) Find the centripetal force provided by the tension in the wire.
- (b) Find the stress in the wire.
4. (a) Calculate the stress in a copper wire of radius 0.20mm when it supports a mass of 2.5kg.
- (b) Calculate the stress in the same wire if the load was doubled to 5.0kg.
- (c) What assumption have you made in calculating (b)?
5. Cast iron has a relatively high proportion of impurity atoms (carbon). It is brittle. Steel has fewer carbon atoms. It is more flexible, and is classed as a strong material. Wrought iron has very few carbon atoms in the lattice. What sort of material would you expect it to be?

Answers

1. $EPE = 0.5 \times (0.20 \times 10^{-2}) \times (60 \times 10^3) = 60J$
 $Heat = (0.3 \times 10^{-2}) \times (65 \times 10^3) = 195J$
2. $\sigma = F/A$, $F = \sigma \times A = (5.0 \times 10^8) \times \pi \times (0.6 \times 10^{-2})^2 = 5.7 \times 10^4 N$
3. (a) $F = \frac{mv^2}{r} = \frac{6.8 \times 6.5^2}{1.65} = 174N$
- (b) $\sigma = \frac{F}{A} = \frac{174}{\pi \times 0.0025^2} = 8.9 \times 10^6 Pa$
4. (a) $\sigma = \frac{F}{A} = \frac{2.5 \times 9.81}{\pi \times (0.20 \times 10^{-3})^2} = 2.0 \times 10^8 Pa$
- (b) $4.0 \times 10^8 Pa$
- (c) We are assuming that the cross-sectional area has not decreased significantly. This probably means that we are operating in the elastic region.
5. You might expect it to be ductile. Movement of crystal planes past each other should be much easier.

Acknowledgements:

This Physics Factsheet was researched and written by Paul Freeman
 The Curriculum Press, Bank House, 105 King Street, Wellington, Shropshire, TF1 1NU
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 ISSN 1351-5136

