Physics Factsheet

www.curriculum-press.co.uk Number 90

Answering Exam Questions: Series and Parallel Calculations

Series and parallel calculations appear not only in electricity (resistors and capacitors), but also in mechanics (elasticity and oscillations).

The first difficulty that occurs is that the relationships involved are not consistent e.g. series and parallel resistor and capacitor systems work in opposite ways. A second difficulty that catches out some students is that these relationships occur within a square-root sign for simple harmonic motion.

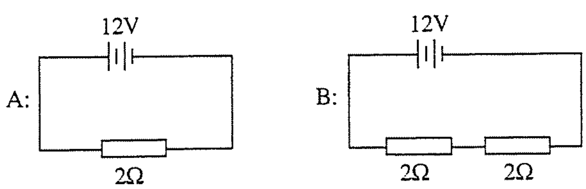
There are other examples of series and parallel relationships such as capacitors and inductors in tuning circuits, but they are beyond the scope of this Factsheet.

Combinations of resistors

Resistors in **series** are simple to deal with. If we add a second resistor in series with an existing one, it is more difficult to push the current through. The effective resistance increases.

Series: $R = R_1 + R_2 + ...$ (ohms, Ω)

Example 1: The diagram shows two circuits, A and B:



Find:
(a) the effective resistance of each circuit.
(b) the current through each circuit.
(c) the power dissipated in each resistor.
(d) the power drawn from each supply.

Answers

(a) A: $R = 2\Omega$
B: $R = 4\Omega$

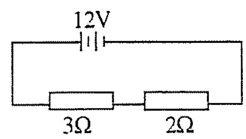
(b) A: $I = \frac{V}{R} = \frac{12}{2} = 6A$
B: $I = 3A$

(c) A: $P = I^2R = 36 \times 2 = 72W$
B: $P = 9 \times 2 = 18W$ for each resistor.

(d) A: supply power, $P = VI = 12 \times 6 = 72W$
B: supply power, $P = 36W$

Adding resistors in series not only reduces the power dissipated in each resistor, it also reduces the power (and current) drawn from the supply.

Example2 : Calculate the power dissipated in each resistor in this circuit.



Answer: $I = \frac{V}{R} = \frac{12}{5} = 2.4A$.

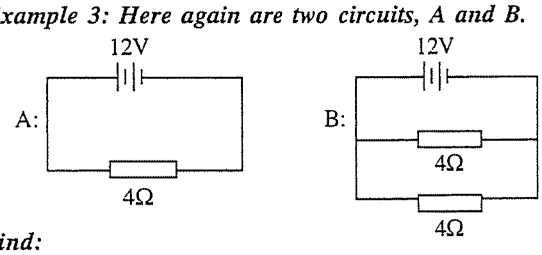
For 3Ω resistor, $P = I^2R = 2.4^2 \times 3 = 17.3W$.
For 2Ω resistor, $P = 11.5W$.

In a series set-up, more power is dissipated in the larger resistor.

Adding resistors in **parallel** increases the possible routes the current can take, making it easier for the current to flow. The effective resistance is smaller than any of the individual resistances.

Parallel: $\frac{1}{R} = \frac{1}{R_1} + \frac{1}{R_2} + ...$ (ohms, Ω)

Example 3: Here again are two circuits, A and B.



Find:
(a) the effective resistance of each circuit.
(b) the current drawn from each supply.
(c) the power dissipated in each resistor.
(d) the power drawn from each supply.

Answers:

(a) A: $R = 4\Omega$
B: $\frac{1}{R} = \frac{1}{4} + \frac{1}{4}$, $R = 2\Omega$

(b) A: $I = \frac{12}{4} = 3A$
B: $I = \frac{12}{2} = 6A$

(c) A: $V = 12V$, $P = \frac{V^2}{R} = \frac{144}{4} = 36W$
B: $P = 36W$ (again) in each resistor

(d) A: $P = VI = 36W$
B: $P = VI = 72W$

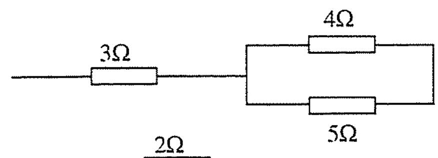
Adding resistors in parallel increases the power (and current) drawn from the supply.

Exam Hints:
(a) Notice from the series and parallel examples that power can be calculated from current through or p.d. across each resistor. Use whichever relationship is most convenient.
(b) The most common error in calculating the effective parallel resistance is forgetting to invert your calculation e.g. $1/R = 1/6$, $R = 6\Omega$.

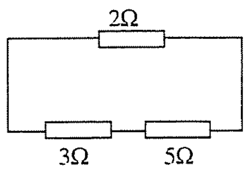
For combinations of series and parallel resistors, just work through logically:

Example 4: Find the effective resistance in each case:

A:



B:



Answer:

(a) 4Ω and 5Ω in parallel: $\frac{1}{R} = \frac{1}{4} + \frac{1}{5}$, $R = 2.2\Omega$

3Ω and 2.2Ω in series, R = 5.2Ω

(b) 3Ω and 5Ω in series: R = 8Ω

2Ω and 8Ω in parallel: R = 1.6Ω

Combinations of capacitors

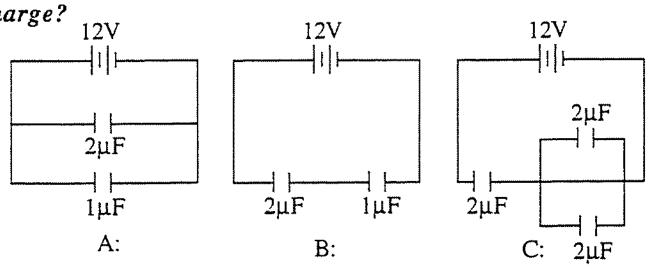
The methods used here are very similar to those for resistors. But remember, capacitors in parallel allow more charge to be stored – the effective capacitance increases. Capacitors in series mean a smaller p.d. across each capacitor, reducing the charge that can be stored – the effective capacitance decreases.

Series: $\frac{1}{C} = \frac{1}{C_1} + \frac{1}{C_2} ...$ (farads, F)

Parallel: $C = C_1 + C_2 + ...$ (farads, F)

Exam hint: *Be very careful with powers. Capacitance is often measured in μF (10^{-6} F).*

Example 5: Which of these combinations would hold the most charge?



Answer:

Circuit A: C = 2 + 1 = 3μF.

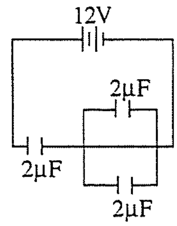
Circuit B: $\frac{1}{C} = \frac{1}{2} + \frac{1}{1} = \frac{3}{2}$, $C = \frac{2}{3}$ *or 0.67μF*

Circuit C: Parallel C = 2 + 2 = 4μF,

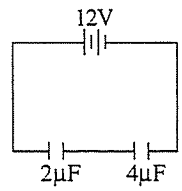
then series $\frac{1}{C} = \frac{1}{2} + \frac{1}{4} = \frac{3}{4}$, *C = 1.33μF.*

So circuit A holds the most charge.

Example 6: Find the energy stored in each capacitor.



Answer: *The parallel combination leads to this equivalent circuit.*



As both capacitors in a series circuit hold the same charge, Q, then from Q = CV, we have

$C_1V_1 = C_2V_2$, *or* $\frac{V_1}{V_2} = \frac{C_2}{C_1} = \frac{4}{2} = \frac{2}{1}$

So the 2μF capacitor has 8 volts across it, and the 4μF capacitor (or the parallel combination) has only 4 volts across it.

The energy stored:
$E_1 = ½ CV^2 = ½ \times 2 \times 10^{-6} \times 64 = 6.4 \times 10^{-5}$ *J.*
Similarly, $E_2 = E_3 = 1.6 \times 10^{-5}$ *J*

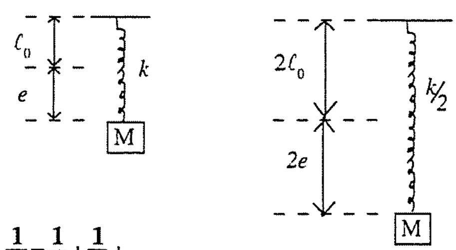
Using the effective capacitance of 1.33μF found earlier, we find the total energy stored is $E = 9.6 \times 10^{-5}$ J (as expected).

Exam hint: *Notice that the total energy stored by the effective capacitance must be the same as that stored by the individual capacitors in total.*

Combinations of springs

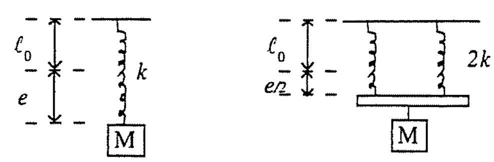
Again, common sense leads you to the correct results. You would expect a long bungee rope to stretch more than a short one. So combining springs in series reduces the stiffness i.e. the **spring constant, k.**

k = F/e (Nm^{-1}), where **e** is the extension of the spring system.



Series: $\frac{1}{\mathbf{k}} = \frac{1}{\mathbf{k_1}} + \frac{1}{\mathbf{k_2}} + ...$

And if you have ever used a muscle building apparatus involving springs, you will know that combining springs in parallel increases the stiffness.



Parallel: $\mathbf{k = k_1 + k_2 + ...}$

Exam hint: *Unlike resistors and capacitors, the springs you might be asked to combine are almost certain to be identical.*

Example:
(a) For three identical springs of spring constant k_o = 400Nm^{-1}, find the effective spring constant when they are combined in series and in parallel.
(b) If each spring has an initial length of 20cm, find the extension in each case if a 30N force is applied to the combination.

Answer:

(a) *In series,* $\frac{1}{k} = \frac{1}{k_o} + \frac{1}{k_o} + \frac{1}{k_o} = \frac{3}{k_o}$, $k = \frac{k_o}{3} = 133\text{Nm}^{-1}$

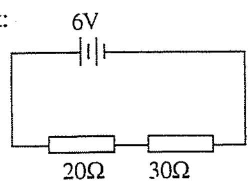
In parallel, $k = k_o + k_o + k_o = 3k_o = 1200\text{Nm}^{-1}$

(b) *In series,* $F = ke$, $e = \frac{F}{k} = \frac{30}{133} = 0.23\text{m}$

In parallel, $e = \frac{30}{1200} = 0.025\text{m}$

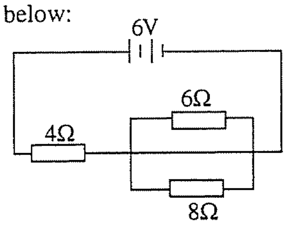
Questions

1. For this circuit:



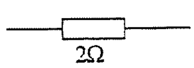
Find
(a) the effective resistance
(b) the current drawn from the supply
(c) the p.d. across each resistor
(d) the power dissipated in each resistor
(e) the power drawn from the supply.

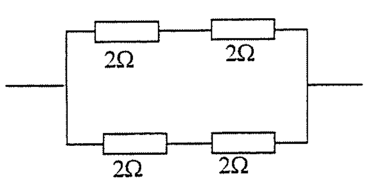
2. For the circuit below:



Find
(a) the effective resistance
(b) the current and power drawn from the supply
(c) the p.d. dropped across each resistor
(d) the power dissipated in each resistor.

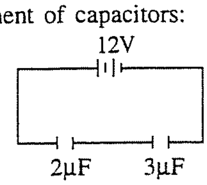
3. A resistance of 2Ω can be constructed from identical resistors in both of these ways:





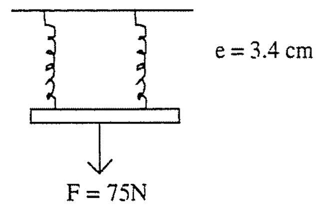
Why might you choose the second arrangement?

4. In this arrangement of capacitors:



(a) Find the effective capacitance and the charge stored in the system.
(b) What is the charge stored on each capacitor?
(c) Find the voltage across each capacitor.
(d) Find the energy stored on each capacitor.
(e) Compare this to the energy provided by the supply. Explain.

5. For this arrangement, find the spring constant for each of these identical springs if a 75N force causes an extension, e, of 3.4cm.



Answers

1 (a) *50Ω.*
(b) $6/50 = 0.12A$
(c) *20Ω:* $V = 0.12 \times 20 = 2.4V$
30Ω: $V = 3.6V$
(d) *20Ω:* $I^2R = 0.29W$
30Ω: 0.43W
(e) $P = VI = 6 \times 0.12 = 0.72W$ $(= 0.29 + 0.43)$

2. (a) $\frac{1}{R_p} = \frac{1}{6} + \frac{1}{8}$, $R_p = 3.4\Omega$
$R = 4 + 3.4 = 7.4\Omega$
(b) $I = 0.81A$, $P = 4.9W$
(c) *4Ω:* $V = IR = 0.81 \times 4 = 3.2V$
Parallel pair: $V = 2.8V$
(d) *4Ω:* $P = \frac{V^2}{R} = 2.6W$
6Ω: $P = \frac{V^2}{R} = 1.3W$
8Ω: $P = 0.98W$
Total = 4.9W as expected.

3. *In a high power circuit, it might be preferable to share the power among the four resistors, to prevent a single resistor overheating.*

4. (a) $\frac{1}{C} = \frac{1}{2 \times 10^{-6}} + \frac{1}{3 \times 10^{-6}}$, $C = 1.2 \times 10^{-6} F$
$Q = CV = 1.44 \times 10^{-5} C.$
(b) *The same in a series circuit:* $Q = 1.44 \times 10^{-5}C$
(c) *2μF:* $V = Q/C = 7.2V$
3μF: $V = 4.8V$
(d) *2μF:* $E = \frac{1}{2}CV^2 = 5.2 \times 10^{-5} J$
3μF: $E = 3.5 \times 10^{-5} J$
(e) *Supply:* $E = QV = 1.73 \times 10^{-4} J$
Total for capacitors, $E = 8.7 \times 10^{-5} J$
Half of the energy drawn from the supply is dissipated in Joule heating.

5. *For the combination,* $k = \frac{F}{e} = \frac{75}{3.4 \times 10^{-2}} = 2205 \text{ Nm}^{-1}$.
Parallel springs, spring constants add.
For each spring, $k = 1103 \text{ Nm}^{-1}$

Acknowledgements:
This Physics Factsheet was researched and written by Paul Freeman
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ISSN 1351-5136