

Physics Factsheet



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The Young Modulus

This Factsheet covers the quantity known as 'the Young modulus' which is used when describing how materials behave when they are stretched by a force. In order to understand the Young modulus we must first consider the quantities of stress and strain, which are used in defining the Young modulus of a material.

Stress

One important factor that affects how a material behaves when it is stretched by a force is the size of the force being used to stretch the material. A bigger force will lead to a bigger stretch. The effect of this stretching force (or 'tensile force') will be different for thick blocks and for thin wires of the same material.

Another important factor to consider when making comparisons between materials is the cross-sectional area of the material being used.

Tensile stress is the force being applied per unit cross sectional area (1 m^2) of the material.

Tensile Stress is defined as:

The force acting on unit cross sectional area of material.

$$\sigma = \frac{F}{A}$$

σ = tensile stress (Pa)
 F = force (N)
 A = cross sectional area (m^2)

The symbol used for tensile stress is the Greek letter σ . Stress has the units Nm^{-2} which is exactly the same as pascals Pa; both units are commonly used.

So, when a force is stretching a material, physicists and engineers will quote the tensile stress that the material is under rather than the force being used, as the tensile stress takes account of the cross sectional area of the material being stretched.

Exam Hint: A common mistake in exam answers is to leave out the words 'cross sectional' when describing the area of the material. These words are important and should always be included in the definition.

Breaking Stress is defined as:

The stress at which a material will break or 'snap'.

Strain

The effect of applying a tensile stress to a material is a stretch, or an extension in length of the material.

The amount of this extension also depends on the original length of the material; a longer piece of material will stretch further than a shorter piece even though they are under the same stress.

The **tensile strain** of a material is the extension per metre of the material.

Tensile strain is defined as:

The extension per unit length of the material.

$$\varepsilon = \frac{e}{l}$$

ε = tensile strain (no units)
 e = force (m)
 l = cross sectional area (m)

The symbol used for strain is the Greek letter ε . Strain has no units as it is calculated by dividing one quantity of length by another quantity of length.

So, when a material is being stretched physicists and engineers will refer to the tensile strain of the material as this takes account of the length of the material.

Exam Hint: When exam questions ask for the definition of tensile stress or tensile strain, the equation can be used as well as the definitions given in words above, but don't forget to define the symbols used in the equations.

Breaking Strain is defined as:

The strain at which a material will break or 'snap'.

Young Modulus

Young Modulus is defined by an equation as:

$$E = \frac{\sigma}{\varepsilon}$$

E = Young modulus (Nm^{-2})
 σ = stress (Nm^{-2})
 ε = strain

The Young modulus has the units of Nm^{-2} , or Pa, the same units as stress. The Young modulus is a measure of the 'stiffness' of the material. A large Young modulus indicates that a large stress is required to produce a small strain and the material does not deform easily. Engineers use the Young modulus for different materials as it does not depend on the dimensions of the material that they are using. It is a quantity for each material that can be used regardless of its shape and size.

Using the equations that have already been introduced for stress and strain, we can substitute into our equation for Young modulus:

$$E = \frac{\sigma}{\varepsilon} = \frac{(F/A)}{(e/l)} = \frac{Fl}{Ae}$$

F = force (N)
 l = original length (m)
 A = cross sectional area (m^2)
 e = extension (m)

This gives us an expression for Young modulus using quantities that can be directly measured from the material being stretched.

Exam Hint: Don't be put off if your values for Young modulus seem large. Typical values for Young modulus vary from rubber, Young modulus = $7 \times 10^5 \text{ Pa}$, to steel, Young modulus = $2 \times 10^{11} \text{ Pa}$.

Typical Exam Question

As part of a quality control check, a manufacturer of washing line subjects a sample to a tensile test. The sample of washing line is 12m long and of constant circular cross section of diameter 5.0mm. The manufacturer measures an extension of 42mm under a stretching load of 72N.

The manufacturer also breaks the line under a load of 240N.

- (a) Calculate the Young modulus of the washing line. [3]
- (b) Calculate the breaking stress of the line. [2]
- (c) If the Young modulus of the line stays constant throughout what is the extension of the line just as it breaks? [3]

Answer

(a) The Young modulus equation requires the cross sectional area of the material so we must first calculate the cross sectional area of the washing line. The cross section of the line is circular and care must be taken as the question gives a measurement for **diameter** in **millimetres**. The measurement must be in metres when used in the equation and the usual equation for areas of a circle requires the radius.

$$A = \pi r^2 = \pi(0.005/2)^2 = 1.96 \times 10^{-5} \text{ m}^2 \quad \checkmark$$

We now have all the figures to substitute into our equation for Young modulus, remembering to change my extension into metres.

$$E = F/Ae = (72)(12)/(1.96 \times 10^{-5})(0.042) = 1.05 \times 10^9 \text{ Pa} \quad \checkmark$$

(b) When calculating the breaking stress we must use the value of force at which the line breaks. We already have a value for the cross sectional area of the line from part (a) of the question.

$$\sigma = F/A = 240/1.96 \times 10^{-5} = 1.22 \times 10^7 \text{ Pa} \quad \checkmark$$

(c) This part of the question is most easily understood if it is completed in two stages. Firstly, knowing that the Young modulus remains unchanged as well as the breaking stress, the breaking strain can be calculated.

$$E = \sigma/\epsilon$$

Rearranging this equation, strain can be made the subject of the equation

$$\epsilon = \sigma/E = 1.22 \times 10^7 / 1.05 \times 10^9 = 0.0117 \quad \checkmark$$

Knowing the breaking strain and the original length the extension can now be calculated.

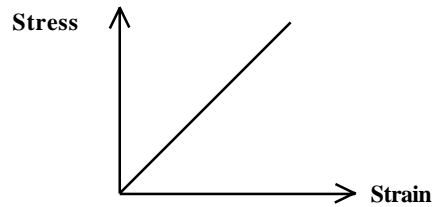
$$\epsilon = e/l$$

Rearranging this equation, extension can be made the subject of the equation.

$$e = \epsilon l = (0.0117)(12) = 0.14 \text{ m} \quad \checkmark$$

Stress – Strain Graphs

For a great many materials, an increase in stress leads to an increase in strain i.e. a bigger stretching force produces a bigger extension. For materials such as metals and glass, if the stress is doubled, then the strain will be doubled also. A graph of stress on the vertical axis against strain on the horizontal axis would therefore be a straight line through the origin.



This graph shows that stress increases at the same rate as strain, or stress is directly proportional to strain.



Hooke's Law

A material that has stress proportional to strain is said to **obey Hooke's Law**. A material that obeys Hooke's law undergoes elastic deformation. This means that when a force is no longer applied to the material it will return to its original shape and will not have any permanent stretch or extension.

The gradient of this graph remains constant and can be calculated by dividing the change in y by the change in x. On the graph you are effectively dividing a stress by a strain. This is the same as calculating the Young modulus of the material.



Calculating Young modulus from a stress – strain graph

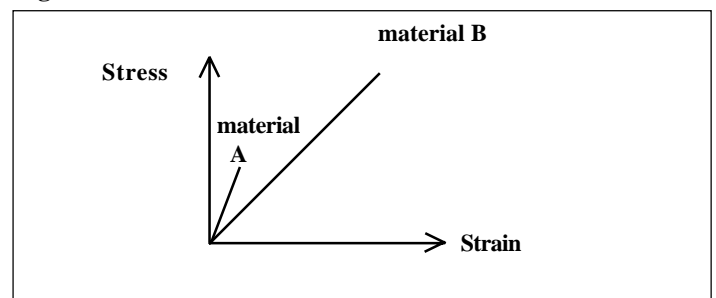
On a stress–strain graph the Young modulus is given by the gradient

$$\text{Young modulus} = \frac{\text{stress}}{\text{strain}} = \text{gradient}$$

The stress – strain graph below has two lines on it for two different materials, A and B. Both of the lines are straight lines through the origin, showing that both materials obey Hooke's law.

- The line for material A has a steeper gradient than the line for material B showing material A to have a larger Young modulus; material A is 'stiffer' than material B.
- Note also how the line for material B finishes higher than material A. This shows that material B has a larger breaking stress than material A; material B is 'stronger' than material A (Fig 1).

Fig 1



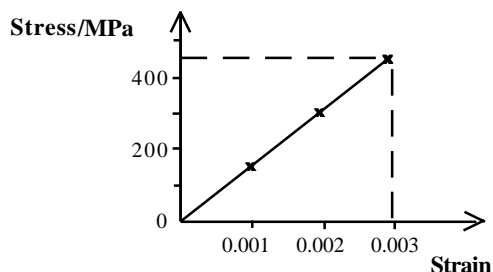
Typical Exam Question

The graph below is the stress-strain graph for a metal that obeys Hooke's law over the region covered by the graph.

(a) What is the Young modulus of the metal? [3]

A wire made from this metal has a diameter of 0.15mm and a length of 2.5m.

(b) Calculate the extension of the wire under stress of 200MPa. [2]



The Young modulus is simply given by the gradient of the graph. Remember that the vertical axis is in MPa.

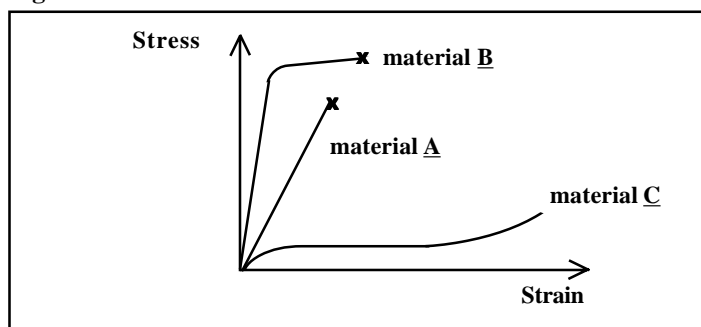
(a) Young modulus, $E = \text{gradient of graph}$
 $= \text{change in height} / \text{change in length}$
 $= 450 \times 10^6 / 0.003 = 1.5 \times 10^{11} \text{ Pa}$ ✓

(b) Reading from the graph, the metal has a strain of 0.0010 when under a stress of 150MPa. ✓

This strain can now be used in the equation for strain: extension, $e = \epsilon l = (0.001)(2.5) = 2.5 \times 10^{-3} \text{ m}$ ✓

Not all stress-strain graphs are straight lines through the origin. Stress-strain graphs can also be curved or a combination of a straight line followed by a curve. The graph below shows three different materials (Fig 3).

Fig 3



Material **A** gives a straight line right up until the material breaks, represented by a cross on the line. This material obeys Hooke's law and breaks very suddenly. The material is brittle. An example of this material is **glass**. Material **B** also obeys Hooke's law initially. It is also stiffer than material **A** as it has a larger Young Modulus represented by the steeper gradient. The Young modulus of material **B** decreases rapidly beyond the point where Hooke's law is obeyed. In this region the material gives a larger strain for increases in stress, this means it deforms easily and is ductile. An example of this material is most **metals**.

Material **C** does not have a constant Young modulus and it is more ductile than material **B**. It also stretches much more before it breaks. An example of this material is rubber.

Key: A brittle material does not behave plastically, it does not change its shape before breaking. A ductile material behaves plastically and will easily change its shape before it breaks.

Exam Workshop

This is a typical poor student's answer to an exam question. The comments explain what is wrong with the answers and how they can be improved. The Examiner's answer is given below.

(a) When determining the Young modulus of a wire, a tensile stress is applied to the wire and the tensile strain produced is measured.

(i) Define the term: 'tensile stress' [2]
 $\text{stress} = \text{force} / \text{area}$ ✓ 1/2

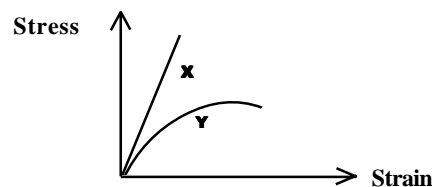
It is important that cross sectional area is specified.

(ii) Define the term: 'tensile strain' [1]
 $\text{Strain} = \text{extension} / \text{length}$ ✓ 1/1

Although this would probably get the mark, original length should be specified. It is also good practice to define both of the above terms in words as well as equations to leave no doubt in the examiner's mind that you should have the marks.

(iii) Define Young modulus [1]
 $\text{Young modulus} = \text{stress} / \text{strain}$ ✓ 1/1

(b) The graph represents stress-strain curves for two different materials X and Y. Both materials are stretched until they break.



(i) Define the term: 'ductile' [2]
Bendy 0/2

This word is not very precise or scientific!

(ii) Which of the materials is ductile? [1]
Material Y ✓ 1/2

(iii) State and explain which of the two material has a greater value of Young modulus. [2]
Material X as it is steeper ✓ 1/2

Although the student has identified the correct material the explanation is difficult to understand and poorly explained.

(c) A fishing line of length 3.0m and diameter 0.20mm supports a load of 25N. Given that the Young modulus for the line is $1.7 \times 10^{10} \text{ Pa}$, calculate the extension in the line produced by this load. [3]

$\text{Area} = \pi r^2 = \pi(0.2)^2 = 0.13$
 $E = FL/Ae, e = FL/AE = (25)(3)/(0.13)(1.7 \times 10^{10}) = 3.4 \times 10^{-8} \text{ m}$ 1/3

The diameter instead of the radius has been used in the calculation of area. Millimetres have not been changed into metres when calculating area.

Examiner's Answers

(a) (i) The force acting on unit cross sectional area of material.

$\text{Tensile stress} = \text{tensile force} / \text{cross sectional area}$.

(ii) The extension per unit length of the material.

$\text{Tensile strain} = \text{extension} / \text{original length}$ ✓

(iii) Young modulus = stress/strain ✓

(b) (i) A ductile material behaves plastically and changes its shape before it breaks.

(ii) Material Y ✓

(iii) Material X has the greatest Young modulus, because the gradient of the stress-strain curve, which represents Young modulus is steepest. ✓

(c) $A = \pi r^2 = \pi(0.1 \times 10^{-3})^2 = 3.14 \times 10^{-8} \text{ m}^2$
 $E = FL/Ae = (25)(3)/(3.14 \times 10^{-8})(1.7 \times 10^{10}) = 0.14 \text{ m}$ ✓

Qualitative Test

1. What is breaking stress?
2. Why do engineers prefer to use quantities such as stress and strain rather than force and extension?
3. How will a strong material differ from a weaker material?
4. How are stress and strain related for a material that obeys Hooke's law? How is this shown on a stress-strain graph?
5. Define a brittle material.
6. How will the stress strain curve for a strong material differ from that of a weaker material. Explain your answer.
7. Describe the stress strain graph of a ductile material.

Quantitative Test

1. A material of diameter 2.0mm supports a load of 60N. Calculate the stress exerted on the material.
2. A material has a breaking strain of 0.075. If a wire has an original length of 0.50m calculate the extension of the material as it breaks.
3. A wire of length 1.5m and diameter 0.25mm extends by 1.5mm when a load of 120N is placed on the wire.
 - (a) Calculate the Young modulus of the wire
 - (b) What extension would a load of 85N produce?
 - (c) What is the strain of the wire with a load of 85N?
4. When certain rocket engines are fired they produce a total thrust force of 4.2×10^6 N. In test firing, the rocket is held to the launch pad by 6 steel bolts, each of diameter 8.0 cm. The Young modulus of steel is 2.0×10^{11} Pa. The breaking stress of steel is 5.0×10^8 Pa.
 - (a) Calculate the strain for each bolt during the test.
 - (b) Determine the minimum number of bolts that could have been used for testing the engines.

Quantitative Test Answers

1. $A = \pi r^2 = \pi(1 \times 10^{-3})^2 = 3.14 \times 10^{-6} \text{ m}^2 \checkmark$
 $\sigma = F/A = 60/(3.14 \times 10^{-6}) = 1.9 \times 10^7 \text{ Pa} \checkmark$
2. $e = \epsilon l = (0.075)(0.5) = 0.0375 \text{ m} \checkmark$
3. (a) $A = \pi r^2 = \pi(0.125 \times 10^{-3})^2 = 4.9 \times 10^{-8} \text{ m}^2 \checkmark$
 $E = Fl/Ae = (120)(1.5)/(4.9 \times 10^{-8})(0.0015) = 2.4 \times 10^{12} \text{ Pa} \checkmark$
 (b) $e = Fl/AE = (85)(1.5)/(4.9 \times 10^{-8})(2.4 \times 10^{12}) = 1.1 \times 10^{-3} \text{ m} \checkmark$
 (c) $\epsilon = e/l = 1.1 \times 10^{-3}/1.5 = 7.3 \times 10^{-4} \checkmark$
4. (a) Area of 1 bolt $= \pi r^2 = \pi(0.04)^2 = 5.03 \times 10^{-3} \text{ m}^2 \checkmark$
 $\sigma = F/A = 4.2 \times 10^6/(6 \times 0.005) = 1.4 \times 10^8 \text{ Pa} \checkmark$
 (b) min number of bolts = total strain/breaking strain per bolt
 $= 5 \times 10^8/1.4 \times 10^8 = 3.57 = 4 \text{ bolts} \checkmark$

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