Physics Factsheet

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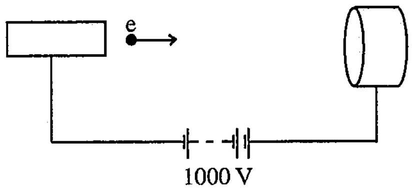
# Calculations on the Quantum Atom

Examiners' reports have highlighted some common errors made by students in attempting energy level calculations. In this Factsheet we shall concentrate on getting the calculations correct rather than developing the theory.

### Joules and the electron-volt

Energy can always be expressed in Joules. But at the atomic level, a Joule is a very large unit for energy.
From the equation **E = QV**, we can see that accelerating a single electron through a p.d. of perhaps 1000 volts would give the electron a kinetic energy:




$KE = QV = 1.6 \times 10^{-19} \times 1000 = 1.6 \times 10^{-16} J$

It is considerably more convenient to just express this as 1000 electron-volts (eV). To convert energy in electron-volts into joules, you just multiply by the electron charge ($1.6 \times 10^{-19}$ C).

> Electron-volts may be convenient, but Joules must be used in all calculations using formulae based on the SI system of units.

***Example:* *A helium nucleus is accelerated from rest through 5000V.***
***(a) Find its kinetic energy in eV***
***(b) Convert this energy into joules***
***(c) Calculate the velocity (using $m = 1.7 \times 10^{-27}$ kg for protons and neutrons)***

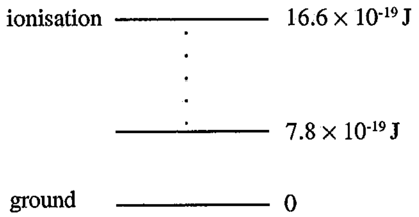
***Answer:***
*(a) $KE = QV = 2 \times 5000 = 10\ 000$ eV, or 10 keV.*
*(b) $KE = 1.6 \times 10^{-19} \times 10\ 000 = 1.6 \times 10^{-15}$ J*
(c) $1.6 \times 10^{-15} = \frac{1}{2}(4 \times 1.7 \times 10^{-27}) \times v^2$, $v = 6.9 \times 10^5\ ms^{-1}$.

***Exam Hint:*** *Be prepared to convert between eV and joules in either direction. To avoid confusion, remember that the value in eV will be a much larger number than the value in joules. In addition be careful to spot prefixes on the unit. It is just as common to write 2.6 MeV as $2.6 \times 10^6$ eV.*

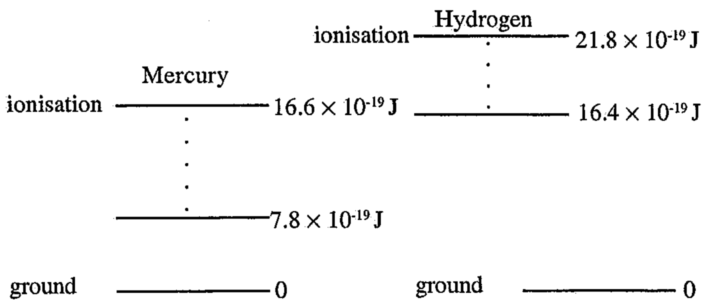
***Example: Write 62 MeV in standard form, then convert to joules.***
***Answer:*** *62 MeV = $6.2 \times 10^7$ eV = $9.9 \times 10^{-12}$J.*

### "Negative" Energy Levels

An atom is in its *ground state* when it cannot lose any energy. However it can gain energy to jump to higher allowed energy states, as its outer electrons become *excited.* It would seem to make sense to call the ground state zero energy, and make higher energy levels positive in value. For a mercury atom

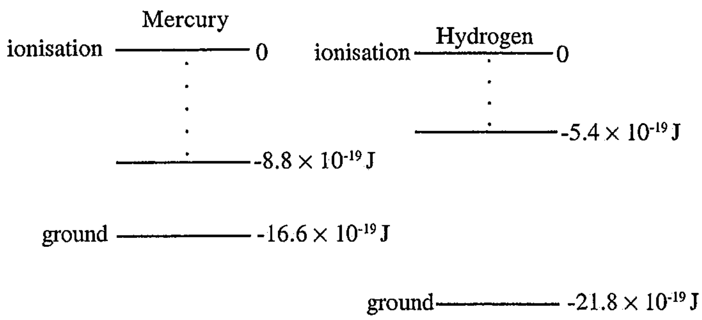



But if we put this next to a hydrogen atom:




This would mean that two identical electrons, freed by ionisation from the two atoms, would appear to have different energies. It wouldn't make sense. The ionisation levels must have the same energy. So we define the ionisation energy levels as zero, and make the lower (trapped) levels negative.

So the mercury and hydrogen atoms energy levels resemble these:




This means that we must be prepared to work with negative numbers.

***Example:***
***(a) Find the energy required to lift an electron in a hydrogen atom from its ground state to the first excited state.***
***(b) Express this in eV.***
***Answer:***
*(a) $-5.4 \times 10^{-19} - (-21.8 \times 10^{-19}) = 16.4 \times 10^{-19}$ J*
*(b) 10.3 eV*

*Exam Hint: Make certain that you know how to work with negative numbers. And check that you know how to enter calculations like that above into your calculator. All calculators do not work in the same way.*

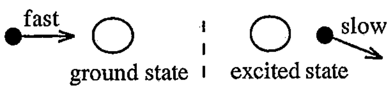
## Transitions between levels

The actual energy of a level is less important than the transitions between levels. You must be prepared to calculate energies needed to excite atoms to higher levels, and also energies emitted (as photons) when atoms drop to lower levels.

***Example: A high-speed electron collides with a hydrogen atom and gives it enough energy to jump from its ground state to its first excited state. What is the minimum speed of this electron? ($m = 9.1 \times 10^{-31}$kg for an electron)***

***Answer:***
*Energy gained = $16.4 \times 10^{-19}$J (as before)*
*$KE = ½mv^2$, $v = 1.9 \times 10^6$ ms$^{-1}$.*
*This is the minimum speed that the electron must have to cause this transition.*

Of course the moving electron mentioned may be going faster than this. If so, it can transfer $16.4 \times 10^{-19}$J to the atom in the collision, and retain the rest of its kinetic energy:




***Example: The high-speed electron strikes the hydrogen atom with a speed of $2.2 \times 10^6$ ms$^{-1}$. If the atom jumps from its ground state to its first excited state, find:***
***(a) the original KE of the electron***
***(b) the KE after the collision***
***(c) its speed after the collision.***

***Answer:***
*(a) $KE = 2.2 \times 10^{-18}$J.*
*(b) $KE = 2.2 \times 10^{-18} - 1.64 \times 10^{-18} = 5.6 \times 10^{-19}$J*
*(c) $v = 1.1 \times 10^6$ms$^{-1}$*

When an incident particle has enough KE to actually *ionise* an atom, the freed electron may well have some KE itself, so we cannot be certain how much KE the incident particle will have remaining.

After an atom is excited into a higher state, it will eventually drop back to its ground state, emitting energy as a quantum of e.m. radiation. The relationship **E = hf = hc/λ** allows us to calculate the wavelength of the radiation emitted. ($h = 6.6 \times 10^{-34}$Js)

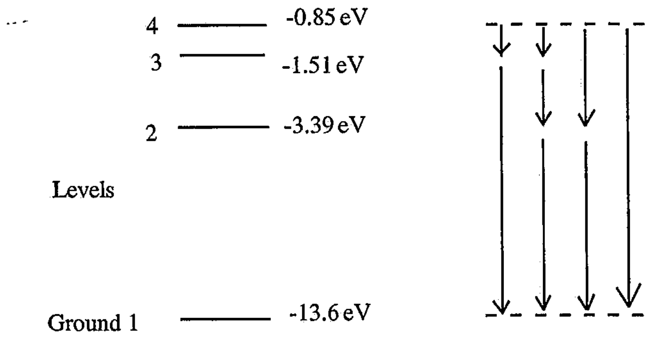
***Example:***
***(a) Find the wavelength emitted by a hydrogen atom when it drops back from its first excited state to its ground state.***
***(b) In which part of the e.m. spectrum is this?***

***Answer:***
*Energy lost = $16.4 \times 10^{-19}$J*
*So $\lambda = hc / E = 1.2 \times 10^{-7}$m. This is in the ultra-violet region.*

***Example: What is the shortest wavelength radiation that could be emitted by a hydrogen atom?***

***Answer:*** *The shortest wavelength implies the greatest energy transition in the atom.*
*This is the drop from the ionisation level to the ground state ($21.8 \times 10^{-19}$J).*
*$\lambda = hc / E = 9.08 \times 10^{-8}$m. This is in the ultraviolet region again.*

Of course an electron can drop down to the ground level in more than one step. This diagram shows the bottom four energy levels of the hydrogen atom (with the energy levels expressed in eV), and possible sets of transitions down to the ground level:




***Example: How many wavelengths of e.m. radiation could be emitted by hydrogen atoms at level 4 returning to the ground state?***
***Answer:*** *There are 6 different energy transitions shown: 4 to 3, 4 to 2, 4 to 1, 3 to 2, 3 to 1, and 2 to 1.*

*Exam Hint: When working out possible sets of transitions, use a logical approach as in the diagram. It is easy to miss one out when putting them in a random order.*

**Practice Questions**

1. An electron has a velocity of $4.0 \times 10^6$ ms$^{-1}$.
   (a) Find the kinetic energy in joules ($m = 9.1 \times 10^{-31}$ kg)
   (b) Find its kinetic energy in eV.
   (c) What p.d. would an electron at rest have to be accelerated through to gain this amount of energy?
2. Write these energies in standard form, then convert them to Joules:
   (a) 3.4 MeV (b) 26.2 keV (c) 95 meV
3. (a) Find the energy required to excite a mercury atom in its ground state to its first excited state.
   (b) Convert this to eV.
   (c) What velocity for a moving electron is matched to this energy?
4. A proton travelling at a speed of 1000ms$^{-1}$ strikes a mercury atom in its ground state. Could it raise the atom into an excited state? ($m = 1.67 \times 10^{-27}$kg)
5. If you were given the bottom five energy levels for an atom, how many different energy transitions are possible?
6. Using the hydrogen atom energy levels given in eV, find the longest wavelength that could be emitted, and identify its position in the e.m. spectrum.

**Answers**

1. (a) $KE = ½mv^2 = 7.28 \times 10^{-18}$J
   (b) KE in eV $= 7.28 \times 10^{-18} / 1.6 \times 10^{-19} = 45.5$ eV.
   (c) p.d. = 45.5 V.
2. (a) 3.4 MeV $= 3.4 \times 10^6$ eV $= 5.4 \times 10^{-13}$ J
   (b) 26.2 keV $= 2.62 \times 10^4$ eV $= 4.2 \times 10^{-15}$ J
   (c) 95 meV $= 9.5 \times 10^{-2}$ eV $= 1.5 \times 10^{-20}$ J
3. (a) $7.8 \times 10^{-19}$ J (b) 4.9 eV (c) $1.3 \times 10^6$ ms$^{-1}$.
4. $KE = ½mv^2 = 8.35 \times 10^{-22}$ joules. This is not nearly sufficient energy.
5. Ten unique transitions.
6. Longest wavelength implies smallest energy transition.
   The smallest energy transition is $-0.85 - (-1.51) = 0.66$ eV $= 1.06 \times 10^{-19}$ J
   So $\lambda = hc / E = 18.6 \times 10^{-7}$ m. This is in the infrared region.

*Acknowledgements:*
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